Department of Systems Engineering & Operation Research
George Mason University

OR 635 Discrete System Simulation

Prof. Chun-Hung Chen Fall 2010

Homework Assignment #5

due 7:20 pm, Tuesday, November 30

****** Due time will be strictly enforced. Late HW is subject to at least 25% penalty ******
****** There is a 20% penalty for the HW turned in through email ******

Definition: **Chen's Favorite Random Numbers** (CFRN) : { 0.025, 0.075, 0.125, 0.175, ..., 0.875, 0.925, 0.975 }. There are 20 numbers in this set. Obviously, these are not good random numbers. However, we will use these numbers several times in our assignments. For your convenience, there is an Excel file containing CFRN available at the class web site.

1. Suppose we conduct 10 independent simulation runs to estimate the mean throughput of a network system. The observed throughputs from these 10 simulation runs are 7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9 and 5.8. Suppose we want to estimate the mean throughput.
   (a) (5 points) What is the 95% confidence interval if we use the Normal distribution model?
   (b) (5 points) What is the 95% confidence interval if we use the t-distribution model?
   (c) (5 points) What is the 99% confidence interval if we use the t-distribution model?

2. Continue Question 1. Again, the observed throughputs from these 10 simulation runs are 7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9 and 5.8. But we want to estimate the probability that the throughput is higher than 8.
   (a) (5 points) What is the 80% confidence interval if we use the Normal distribution model?
   (b) (5 points) What is the 80% confidence interval if we use the t-distribution model?

3. In this question, we want to develop a simulator for planning your investment. A similar simulator developed by T. Rowe Price can be accessed vie our class web site. Consider the following two options to invest your money:

I. CD: low return rate without any risk.

II. Stock: high return rate with high risk

Based on historical data analysis, we have found that the yearly return rate for CD is 6%. Due to the risk of stocks, the yearly return rates are not guaranteed, but can be represented by statistical distribution: Uniform(-40%, 60%). This means that you may lose your money if you buy stocks, although the return rate can be as high as 60% per year. Suppose you have $1,000,000 for investment. Develop a Monte Carlo simulator to evaluate the investment for a 4-year period.

(a) (5 points) What is the expected total after 4 years if you invest all of your money in CD?

(b) (10 points) For stock, use CFRN to conduct 5 simulation runs. In each run, please plot a curve which indicates the total money you will have at the end of each year. Your end result will be 5 curves something like the picture shown the above T. Rowe Price web site. In this case, you need four Unif(0,1) random numbers in one run. Use the 1st, 2nd, 3rd, and 4th numbers (i.e., 0.025, 0.075, 0.125, 0.175) to generate the four random numbers for first run. Then you use 5th~8th numbers for next run, etc. Thus you will be able to have 5 simulation runs using the 20 CFRN numbers. Please turn in the figure showing the 5 curves. In addition, please estimate the expected total and estimate the probability that your total is less than $500,000 at the end of the 4-year period, if you invest all of your money in stocks.

(c) (10 points) Repeat part (b). Instead of using CFRN, use the build-in function rand( ) in Excel and have 100 simulation runs (so you need 400 random numbers). Specifically, You have to i) Plot the 100 curves which indicates the total money you will have over the period of 4 years for the 100 simulation runs; ii) Estimate the expected total if you invest all of your money in stocks; and iii) Estimate the probability that your expected total is less than $500,000 at the end of the 4-year period.

4. We plan to run a specific projector for 99 hours in an exhibition. The lifetime of the light bulb used in this projector is a main concern because it is relatively short and has some uncertainty. There are three brands of bulbs for consideration: A, B, and C. Based on the specifications from their manufacturers, the lifetimes for a bulb for brands A, B, and C are EXP(0.04), EXP(0.1), and Normal(25, 5^2), respectively. The purchasing costs of one bulb for brands A, B, and C are \$25, \$12, and \$33, respectively. The constraint is that you have to purchase the bulbs before going to the exhibition and bring them with you. Ultimately we want the ***total*** lifetime of the bulbs we bring can sustain longer than the required 99 hours while the ***total*** cost of the bulbs is minimized. Assume that the lifetime of each bulb is independent. Develop a Monte Carlo simulator using CFRN for the following questions. To generate normal random variates, please use the NORMINV function in Excel.

(a) (10 points) If you decide to bring 3 A bulbs and 1 C bulb, what is the expected total lifetime? What is the 99% confidence interval for this estimation using the *t* model? Use the 1st, 2nd, 3rd, and 4th numbers (i.e., 0.025, 0.075, 0.125, 0.175) to generate the four random numbers in the first run. Then you use 5th~8th numbers for next run, etc. Thus you will be able to have 5 simulation runs using the 20 CFRN numbers.

(b) (10 points) Continue Part (a). What is the probability that their total lifetime exceeds 99 hours? What is the 99% confidence interval for this estimation using the *t* model?